Theor Appl Genet (1988) 76:465–466

Further approximations for selection intensity*

A. M. Saxton

Department of Experimental Statistics, Louisiana Agricultural Experiment Station, Louisiana State University Agricultural Center, Baton Rouge, LA 70803, USA

Received January 16, 1987; Accepted April 28, 1987
Communicated by E. J. Eisen

Summary. Three new approximations are suggested for the standardized selection intensity, i. Two are simple functions of powers of b, the fraction selected. These improve on previous approximations by covering a broader range of selection intensities. A third approximation is developed using a rational polynomial. This gave accurate approximation, but simplicity was lost.

Key words: Computer approximation – Rational polynomials.

Introduction

As a standardized measure of expected selection pressure, selection intensity, i, plays an important role in quantitative genetics. Typically, a certain fraction (b) of the population is to be selected, and from this value i must be calculated. Tables for this task are available (Falconer 1965, Becker 1984), but as pointed out by Simmonds (1977), use of tables is not always feasible. Assuming a normal and infinite population, selection intensity can be calculated as

$$i = \exp(-z^2/2)/(b\sqrt{2\pi}).$$

Here z is used to represent the point on the standard normal curve above (assuming positive selection) which a fraction b of the population lies, and can be looked up in tables. If tables are to be avoided, inverse normal computer algorithms can be used (such as Beasley and Springer 1977), which compute z for a given b.

Several researchers have expressed a need for a simple approximation for i. The approximation of Smith (1969) (Table 1) was felt to be too limited in range by Simmonds (1977), who suggested two more functions for the cases of very low and very high selection intensity. The purpose of this paper is to suggest single functions that cover the same range of selection intensities. An approximate correction for finite or non-normal populations is given by Burrows (1972).

Methods

Due to the pronounced non-linearity of the intensity function (Fig. 1), a power transformation of b was used. By trial and error, raising b to the 0.2 power was found to result in near linearity. Figure 1 shows that slight curvature remains at extreme selection intensities. A search for accurate approximations was then made, examining various functions of powers of b.

Rational polynomials are widely used to approximate functions. An excellent discussion and examples are given in Moshier (1986). Basically, the idea is to approximate i with a ratio of two polynomials in $b^{0.2}$, for example

$$\frac{n_0 + n_1 b^{0.2} + n_2 b^{0.4} + n_3 b^{0.6}}{d_0 + d_1 b^{0.2} + d_2 b^{0.4}}.$$

Methods discussed by Moshier (1986) were used to compute numerator (n_j) and denominator (d_j) constants that approximate i.

Evaluation of functions (Table 1) was done by comparing "exact" and approximate i at values of b between 10^{-5} and 0.01 in multiplicative steps of $10^{0.2}$, from 0.01 to 0.99 in additive steps of 0.01, and the values 0.999, 0.9999 and 1.0. At each point, absolute and percentage errors were calculated.

Results and discussion

Three functions were developed, ranging from a relatively inaccurate simple regression to an accurate but

* Approved for publication by the Director of the Louisiana Agricultural Experiment Station as manuscript no. 87-19-10038.

Table 1. Comparison of maximum percentage error (corresponding absolute error in parentheses) among various approximations for i; since i = 0 for b = 1, percentage error is not given for this case

Range in b	Function					
	A	B	C	D	E	F
10^{-5} – 10^{-4}	–	7 (0.29)	–	9 (0.40)	11 (0.47)	0.5 (0.02)
10^{-4} – 0.004	–	3 (0.08)	–	4 (0.11)	5 (0.20)	0.5 (0.02)
0.006 – 0.1	3 (0.06)	5 (0.09)	–	4 (0.11)	4 (0.09)	0.6 (0.01)
0.11 – 0.19	3 (0.05)	–	–	2 (0.03)	2 (0.03)	0.4 (0.005)
0.20 – 0.75	17 (0.07)	–	5 (0.04)	8 (0.06)	2 (0.02)	0.5 (0.002)
0.76 – 0.90	–	–	13 (0.03)	35 (0.07)	1 (0.003)	0.5 (0.002)
1.00	–	–	(0.002)	(0.17)	(0.006)	(0.003)

A	Smith (1969)	$0.8 + 0.41 \ln (1/b - 1)$
B	Simmonds (1977)	$1.132 + 0.729 \log (1/b)$
C	Simmonds (1977)	$1.672 - 1.670\, b$
D		$4.5122 - 4.3382\, b^{0.2}$
E		$4.4206 - 4.1683\, b^{0.2} - 0.246\, b^{5}$
F		$\dfrac{2.97425 - 3.35197\, b^{0.2} - 1.9319\, b^{0.4} + 2.3097\, b^{0.6}}{0.51953 + 0.88768\, b^{0.2} - 2.38388\, b^{0.4} + b^{0.6}}$

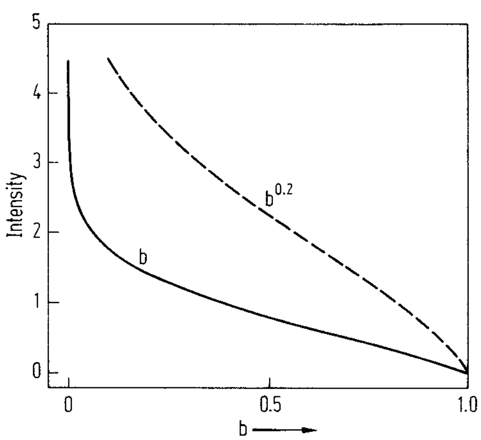

Fig. 1. The effect of power transformation on the relationship between b and i

complex rational polynomial. The two simple functions proposed here (D and E in Table 1) are similar in accuracy to previous approximations and have the advantage of covering a wide range of selection intensities. By adding in the fifth power of b, accuracy of the approximation was improved greatly for low selection intensities.

The rational polynomial is given to show the complexity required to obtain an approximation with consistently high accuracy. An advantage of rational polynomials is that numerator and denominator constants can be easily manipulated to yield equal percentage error throughout the range of interest, as was done here, or could be changed to give higher accuracy for particular selection intensities. In contrast, a fourier series approximation (not shown), using six terms involving sines and cosines of $b^{0.2}$, gave high accuracy as measured by the sum of squared deviations from a least squares fit. But the percentage error could not be controlled, and was as high as 5% for certain b. Since the number of terms required by the fourier series was similar to the rational polynomial, this approach was not pursued.

The rational polynomial given in Table 1 may be too complex for occasional use, but could be used as the basis for a computer algorithm. In fact, the algorithm of Beasley and Springer (1977) mentioned above is based on a rational polynomial. However, if a computer is used, the latter algorithm which gives "exact" results should be considered. The rational polynomial could be made more accurate by adding terms to the numerator and denominator. Simmonds (1977) argues that extreme accuracy is not needed in practice, as the accuracy of heritability estimates and other parameters of genetic interest will generally be the limiting factor in computing genetic gain, the main use of i.

References

Beasley JD, Springer SG (1977) Algorithm AS 111: The percentage points of the normal distribution. Appl Stat 26: 118–121

Becker WA (1984) Manual of quantitative genetics, 4 th edn. Academic Enterprises, Pullman/WA

Burrows PM (1972) Expected selection differentials for directional selection. Biometrics 28: 1091–1100

Falconer DS (1965) The inheritance of liability to certain diseases, estimated from the incidence among relatives. Ann Human Genet 29: 51–76

Moshier SL (1986) Computer approximations. Byte 11: 161–178

Simmonds NW (1977) Approximations for i, intensity of selection. Heredity 38: 413–414

Smith C (1969) Optimum selection procedures in animal breeding. Anim Prod 11: 433–442